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Sturm's Theorems for Canonical Equation of the Second Order with Non-monotonous Coefficients

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Abstract. For canonical equation $y'' + a(x) \cdot y = 0$, if a(x) is strictly monotonous function, we have determined number of zeros and exact locations of zero solutions which are main weak points of classic Sturm's theorems. However, if in equation, $y'' = -a(x) \cdot y$, a(x) is non-monotonous function, then function $F(x, y) = -a(x) \cdot y$ is not monotonous, as well and its inverse function is the multi-valued one. This naturally influences both oscillarity and zero oscillations. Therefore, this case has been considered in this work. When the coefficient a(x) is non-monotonous function, we have first determined all intervals of monotony of coefficient. Then in each interval, we have determined number of zeros and very precise locations of zeros, linearly particular solutions y_1 and y_2 and thus improved Sturm's theorems, which are almost 180 years old.

Keywords: Differential equations, iterations, oscillating solutions, locations of zero oscillations, non-monotonous coefficient, amplitudes, frequencies

1. Introduction

Let us have canonical differential equation